PARCIÁLNÍ DERIVACE – příklad 1

$$f(x,y) = xy \cdot \ln(2x+3y) - \ln 5$$

1) $\frac{\partial f}{\partial x} = (y \text{ je konstanta}) = y \cdot \ln(2x+3y) + x \cdot y \cdot \frac{1}{2x+3y} \cdot (2+0) =$

$$= y \cdot \ln(2x+3y) + \frac{2xy}{2x+3y}$$

2) $\frac{\partial^2 f}{\partial x \partial y} = (\text{zderivujeme } \frac{\partial f}{\partial x} \text{ podle } y;\ x \text{ je konstanta}) =$

$$= \ln(2x+3y) + y \cdot \frac{1}{2x+3y} \cdot (0+3) + \frac{2x(2x+3y) - 2xy \cdot (0+3)}{(2x+3y)^2} =$$

$$= \ln(2x+3y) + \frac{3y}{2x+3y} + \frac{4x^2 + \cancel{6xy} - \cancel{6xy}}{(2x+3y)^2} = \ln(2x+3y) + \frac{3y(2x+3y) + 4x^2}{(2x+3y)^2}$$

$$= \ln(2x+3y) + \frac{6xy + 9y^2 + 4x^2}{(2x+3y)^2}$$

3) $\frac{\partial f}{\partial y} = (\text{vracíme se k zadané funkci, } x \text{ je konstanta}) =$

$$= x \cdot \ln(2x+3y) + \frac{xy}{2x+3y} \cdot (0+3) = x \ln(2x+3y) + \frac{3xy}{2x+3y}$$

4) $\frac{\partial^2 f}{\partial y \partial x} = (\text{zderivujeme } \frac{\partial f}{\partial y} \text{ podle } x,\ y \text{ je konstanta}) =$

$$= \ln(2x+3y) + x \cdot \frac{1}{2x+3y} \cdot (2+0) + \frac{3y(2x+3y) - 3xy(2+0)}{(2x+3y)^2} =$$

$$= \ln(2x+3y) + \frac{2x}{2x+3y} + \frac{\cancel{6xy} + 9y^2 - \cancel{6xy}}{(2x+3y)^2} = \ln(2x+3y) + \frac{2x(2x+3y) + 9y^2}{(2x+3y)^2} =$$

$$= \ln(2x+3y) + \frac{4x^2 + 6xy + 9y^2}{(2x+3y)^2}$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$