Taylorův polynom 3. stupně

$$f(x) = x^2 + 2 - \sqrt{1-x} \qquad x = 0$$

I) Dopočtení souřadnice bodu, ve kterém je třeba spočítat T. p.

$$f(0) = 0^2 + 2 - \sqrt{1-0} = 2 - 1 = \underline{\underline{1}} \qquad [0,1]$$

II) 1. derivace

$$f'(x) = 2x - \frac{1}{2\sqrt{1-x}} \cdot (-1) = 2x + \frac{1}{2\sqrt{1-x}}$$

$$f'(0) = 2 \cdot 0 + \frac{1}{2 \cdot \sqrt{1}} = \underline{\frac{1}{2}}$$

III) 2. derivace

$$f''(x) = 2 + \frac{0 \cdot 2\sqrt{1-x} + 1 \cdot \cancel{2} \cdot \frac{1}{\cancel{2} \cdot \sqrt{1-x}} \cdot (+1)}{(2\sqrt{1-x})^2} = 2 + \frac{\frac{1}{\sqrt{1-x}}}{4(1-x)} =$$

$$= 2 + \frac{1}{\sqrt{1-x}} \cdot \frac{1}{4(1-x)} = 2 + \frac{1}{4(1-x)\sqrt{1-x}} = 2 + \frac{1}{4\sqrt{(1-x)^3}} = 2 + \frac{1}{4(1-x)^{\frac{3}{2}}}$$

$$f''(0) = 2 + \frac{1}{4 \cdot 1} = 2 + \frac{1}{4} = \frac{8+1}{4} = \underline{\underline{\frac{9}{4}}}$$

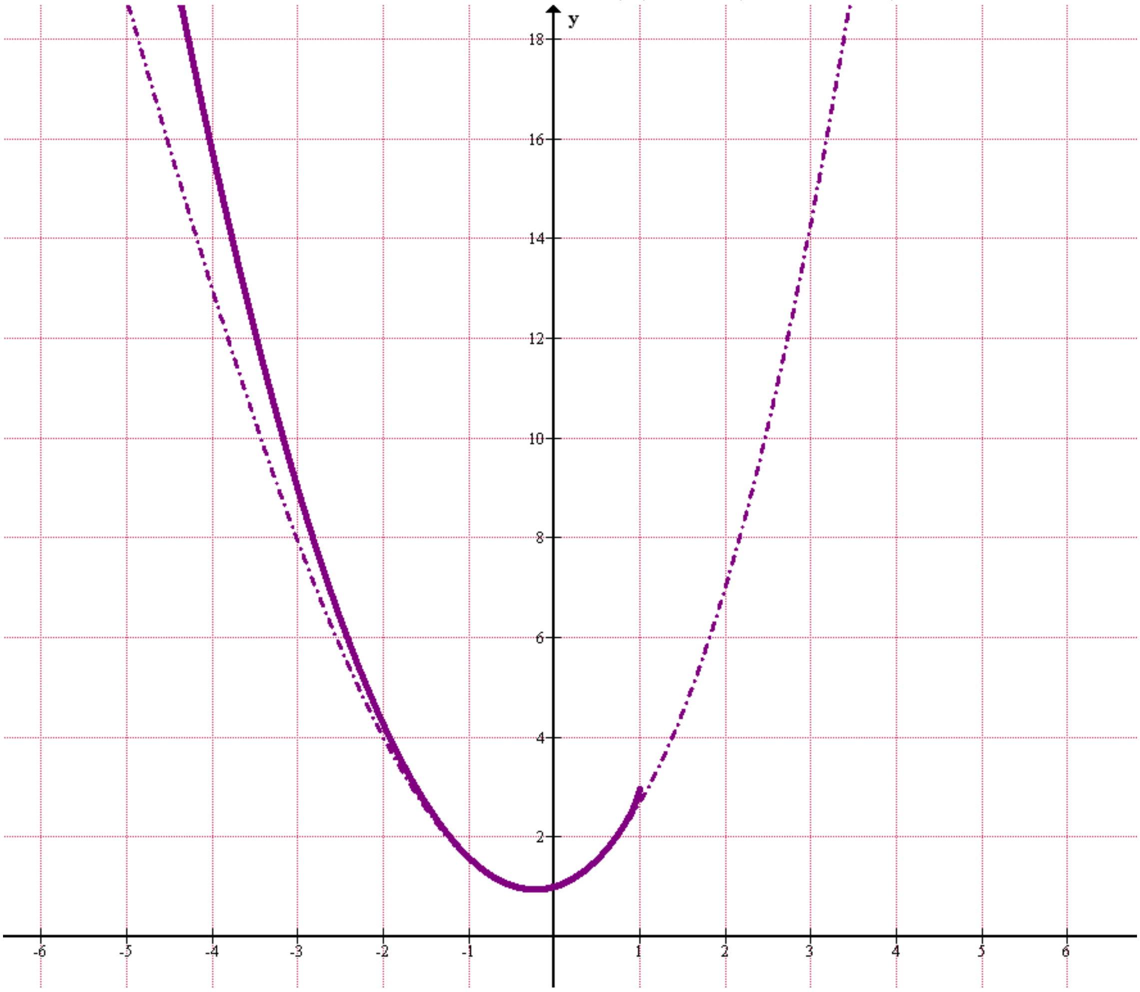
IV) 3. derivace

$$f'''(x) = \frac{0 - 1 \cdot 4 \cdot \frac{3}{2}(1-x)^{\frac{1}{2}} \cdot (-1)}{[4(1-x)\sqrt{1-x}]^2} = \frac{2 \cdot 3\sqrt{1-x}}{16(1-x)^2 \cdot (1-x)} = \frac{6\sqrt{1-x}}{16(1-x)^3} = \frac{3\sqrt{1-x}}{8(1-x)^3}$$

$$f'''(0) = \frac{3}{8}$$

$$\underline{T_3(x) = 1 + \frac{1}{2}x + \frac{9}{8}x^2 + \frac{1}{16}x^3}$$

Obrázek 1. Grafické znázornění funkce a jejího Taylorova polynomu 3. stupně



Zdroj: program Graph