Zkouškové: 20)

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$$y = -\frac{\arcsin x}{x} + \ln \frac{(1-\sqrt{1-x^2})^{\alpha}}{x^{\alpha}}$$

$$= \ln \frac{1-\sqrt{1-x^2}}{x} \ominus \frac{\arcsin x}{x}$$

$$y' = \frac{1}{\frac{1-\sqrt{1-x^2}}{x}} \cdot \frac{\left[0 + \frac{1}{2\sqrt{1-x^2}} \cdot (-2x)\right] x - (1-\sqrt{1-x^2}) \cdot 1}{x^2} \ominus$$

$$\ominus \frac{\frac{1}{\sqrt{1-x^2}} \cdot x - \arcsin x \cdot 1}{x^2} =$$

$$= \frac{x}{1-\sqrt{1-x^2}} \cdot \frac{\frac{x^2}{\sqrt{1-x^2}} - 1 + \sqrt{1-x^2}}{x^2} - \frac{\frac{x}{\sqrt{1-x^2}} - \arcsin x}{x^2} =$$

$$= \frac{1}{1-\sqrt{1-x^2}} \cdot \left(\frac{\frac{x^2}{\sqrt{1-x^2}}}{\frac{x}{1}} - \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right) - \frac{\frac{x}{\sqrt{1-x^2}}}{x^2} - \frac{\arcsin x}{x^2} =$$

$$= \frac{1}{1-\sqrt{1-x^2}} \cdot \left(\frac{x}{\sqrt{1-x^2}} - \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right) - \frac{1}{x\sqrt{1-x^2}} - \frac{\arcsin x}{x^2} =$$

$$= \frac{x}{\sqrt{1-x^2}(1-\sqrt{1-x^2})} - \frac{1}{x(1-\sqrt{1-x^2})} + \frac{\sqrt{1-x^2}}{x(1-\sqrt{1-x^2})} - \frac{1}{x\sqrt{1-x^2}} - \frac{\arcsin x}{x^2} =$$

$$= \frac{x}{\sqrt{1-x^2} \cdot (1-\sqrt{1-x^2})} + \frac{\sqrt{1-x^2}-1}{x(1-\sqrt{1-x^2})} - \frac{x - x\sqrt{1-x^2} \cdot \arcsin x}{x^2\sqrt{1-x^2}} =$$

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$$= \frac{x \cdot x\left(1-\sqrt{1-x^2}\right)+\left(\sqrt{1-x^2}-1\right)\cdot\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)}{\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)\cdot x\left(1-\sqrt{1-x^2}\right)} -$$

$$- \frac{x\left(1-\sqrt{1-x^2}\cdot \arcsin x\right)}{x^2\sqrt{1-x^2}} =$$

$$= \frac{\left(1-\sqrt{1-x^2}\right)\cdot\left[x^2+\left(\sqrt{1-x^2}-1\right)\sqrt{1-x^2}\right]}{\sqrt{1-x^2}\cdot x\left(1-\sqrt{1-x^2}\right)^2} - \frac{1-\sqrt{1-x^2}\cdot\arcsin x}{x\sqrt{1-x^2}} =$$

$$= \frac{x^2+\sqrt{1-x^2}\cdot\left(\sqrt{1-x^2}-1\right)}{x\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)} - \frac{1-\sqrt{1-x^2}\cdot\arcsin x}{x\sqrt{1-x^2}} =$$

$$= \frac{x^2+(1-x^2)-\sqrt{1-x^2}-\left[\left(1-\sqrt{1-x^2}\cdot\arcsin x\right)\cdot\left(1-\sqrt{1-x^2}\right)\right]}{x\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)} =$$

$$= \frac{x^2+1-x^2-\sqrt{1-x^2}-\left[1-\sqrt{1-x^2}-\sqrt{1-x^2}\cdot\arcsin x+(1-x^2)\arcsin x\right]}{x\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)} =$$

$$= \frac{1-\sqrt{1-x^2}-1+\sqrt{1-x^2}+\sqrt{1-x^2}\cdot\arcsin x-(1-x^2)\arcsin x}{x\sqrt{1-x^2}\cdot\left(1-\sqrt{1-x^2}\right)} =$$

$$= \frac{\sqrt{1-x^2}\cdot\arcsin x-\arcsin x+x^2\cdot\arcsin x}{x\left(\sqrt{1-x^2}-(1-x^2)\right)} =$$

$$= \frac{\arcsin x\left(\sqrt{1-x^2}-1+x^2\right)}{x\left(\sqrt{1-x^2}-1+x^2\right)} = \frac{\arcsin x}{x}$$