

$$1) \int x \cdot \cos x \, dx = \left| \begin{array}{ll} u = \cos x & v = x \\ u' = -\sin x & v' = 1 \end{array} \right| = x \cdot \sin x - \int 1 \cdot \sin x \, dx =$$

$$= x \cdot \sin x + \cos x + C$$

$$2) \int x^3 \cdot \ln x \, dx = \left| \begin{array}{ll} u' = x^3 & v = \ln x \\ u = \frac{x^4}{4} & v' = \frac{1}{x} \end{array} \right| = \frac{x^4}{4} \ln x - \int \frac{x^4}{4} \cdot \frac{1}{x} \, dx =$$

$$= \frac{x^4}{4} \ln x - \frac{1}{4} \int x^3 \, dx = \frac{x^4}{4} \ln x - \frac{1}{4} \cdot \frac{x^4}{4} + C = \frac{x^4}{4} \left(\ln x - \frac{1}{4} \right) + C$$

$$3) \int \ln x \, dx = \left| \begin{array}{ll} u' = 1 & v = \ln x \\ u = x & v' = \frac{1}{x} \end{array} \right| = x \ln x - \int x \cdot \frac{1}{x} \, dx = x \cdot \ln x - x + C$$

$$4) \int x^2 \cdot e^x \, dx = \left| \begin{array}{ll} u' = e^x & v = x^2 \\ u = e^x & v' = 2x \end{array} \right| = x^2 e^x - \int 2x \cdot e^x \, dx =$$

$$= x^2 \cdot e^x - \left| \begin{array}{ll} u' = e^x & v = 2x \\ u = e^x & v' = 2 \end{array} \right| = x^2 e^x - [2x e^x - \int 2e^x \, dx] =$$

$$= x^2 e^x - 2x e^x + 2e^x + C$$

$$5) \int \sin x \cdot e^x dx = \left| \begin{array}{ll} u' = e^x & v = \sin x \\ u = e^x & v' = \cos x \end{array} \right| = e^x \cdot \sin x - \int e^x \cdot \cos x dx =$$

$$= e^x \cdot \sin x - \left| \begin{array}{ll} u' = e^x & v = \cos x \\ u = e^x & v' = -\sin x \end{array} \right| = e^x \cdot \sin x - \left[e^x \cdot \cos x + \int \sin x \cdot e^x dx \right] =$$

Tento typ integrálu "vede na rovnici":

$$\underbrace{\int \sin x \cdot e^x dx}_{\text{řadím}} = \underbrace{e^x \cdot \sin x - e^x \cdot \cos x - \int \sin x \cdot e^x dx}_{\text{dosadím výsledek}} \quad + \int \sin x \cdot e^x dx$$

$$2 \int \sin x \cdot e^x dx = e^x \cdot \sin x - e^x \cdot \cos x \quad |:2$$

$$\int \sin x \cdot e^x dx = \frac{e^x \cdot \sin x - e^x \cdot \cos x}{2} + C$$

$$6) \int x^8 \cdot \ln x dx = \left| \begin{array}{ll} u' = x^7 & v = \ln x \\ u = \frac{x^8}{8} & v' = \frac{1}{x} \end{array} \right| = \frac{x^8}{8} \ln x - \int \frac{x^8}{8} \cdot \frac{1}{x} dx =$$

$$= \frac{x^8}{8} \ln x - \frac{1}{8} \int x^7 dx = \frac{x^8}{8} \ln x - \frac{1}{8} \cdot \frac{x^8}{8} + C = \frac{x^8}{8} \cdot \left(\ln x - \frac{1}{8} \right) + C$$

$$7) \int x \cdot 2^x dx = \left| \begin{array}{ll} u' = 2^x & v = x \\ u = \frac{2^x}{\ln 2} & v' = 1 \end{array} \right| = x \cdot \frac{2^x}{\ln 2} - \int \frac{2^x}{\ln 2} dx = \frac{x 2^x}{\ln 2} - \frac{1}{\ln 2} \int 2^x dx =$$

$$= \frac{x \cdot 2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \frac{2^x}{\ln 2} + C = \frac{2^x}{\ln 2} \cdot \left(x - \frac{1}{\ln 2} \right) + C$$

$$8) \int (x^3 + 2x^2 - 5x) \cdot e^x dx = \left| \begin{array}{ll} u' = e^x & v = x^3 + 2x^2 - 5x \\ u = e^x & v' = 3x^2 + 4x - 5 \end{array} \right| =$$

$$= (x^3 + 2x^2 - 5x)e^x - \int (3x^2 + 4x - 5)e^x dx = (x^3 + 2x^2 - 5x)e^x - \left| \begin{array}{ll} u' = e^x & v = 3x^2 + 4x - 5 \\ u = e^x & v' = 6x + 4 \end{array} \right| =$$

$$= (x^3 + 2x^2 - 5x)e^x - \left[(3x^2 + 4x - 5)e^x - \int (6x + 4)e^x dx \right] =$$

$$= (x^3 + 2x^2 - 5x)e^x - (3x^2 + 4x - 5)e^x + \left| \begin{array}{ll} u' = e^x & v = 6x + 4 \\ u = e^x & v' = 6 \end{array} \right| =$$

$$= (x^3 + 2x^2 - 5x)e^x - (3x^2 + 4x - 5)e^x + \left[(6x + 4)e^x - 6 \int e^x dx \right] =$$

$$= (x^3 + 2x^2 - 5x)e^x - (3x^2 + 4x - 5)e^x + (6x + 4)e^x - 6e^x + C =$$

$$= \underbrace{e^x}_{\text{vytáeno}} (x^3 + 2x^2 - 5x - 3x^2 - 4x + 5 + 6x + 4 - 6) + C =$$

$$= e^x (x^3 - x^2 - 3x + 3) + C$$

$$9) \int (x+3) \cos x dx = \left| \begin{array}{ll} u' = \cos x & v = x+3 \\ u = \sin x & v' = 1 \end{array} \right| = (x+3) \sin x - \int \sin x dx =$$

$$= \sin x \cdot (x+3) + \cos x + C$$

$$10) \int x \cdot \arctg x dx = \left| \begin{array}{ll} u' = x & v = \arctg x \\ u = \frac{x^2+1}{2} & v' = \frac{1}{1+x^2} \end{array} \right| = \frac{x^2+1}{2} \arctg x - \int \frac{x^2+1}{2} \cdot \frac{1}{1+x^2} dx =$$

$$= \frac{x^2+1}{2} \arctg x - \frac{1}{2} x + C$$

* Po integraci se vždy přičítá +C. V pomoci tabulky a metody per partes to nikdy neděláme, nicméně lze přičít konstantu. A to i jako konkrétní vybranou hodnotu.

$$11) \int (5-5x) \cos x \cdot dx = \left| \begin{array}{ll} u' = \cos x & v = 5-5x \\ u = \sin x & v' = -5 \end{array} \right| = (5-5x) \cdot \sin x + 5 \int \sin x dx =$$

$$= (5-5x) \sin x - 5 \cos x + C$$

$$12) \int 3^x \cdot \cos x \cdot dx = \left| \begin{array}{ll} u' = \cos x & v = 3^x \\ u = \sin x & v' = 3^x \cdot \ln 3 \end{array} \right| = 3^x \cdot \sin x - \ln 3 \int 3^x \cdot \sin x dx =$$

$$= 3^x \cdot \sin x - \ln 3 \left| \begin{array}{ll} u' = \sin x & v = 3^x \\ u = -\cos x & v' = 3^x \cdot \ln 3 \end{array} \right| = 3^x \cdot \sin x - \ln 3 \left[-3^x \cdot \cos x + \ln 3 \int 3^x \cdot \cos x dx \right] =$$

Tento příklad 'vede na rovnici':

$$3^x \cdot \sin x + 3^x \ln 3 \cos x - \ln^2 3 \int 3^x \cdot \cos x dx = \int 3^x \cdot \cos x dx$$

dosavadní výsledek zadání

$$\int 3^x \cdot \cos x dx = \frac{3^x \sin x + 3^x \ln 3 \cdot \cos x}{1 + \ln^2 3} + C$$

$$13) \int \frac{\ln x}{\sqrt{x}} dx = \int \ln x \cdot \frac{1}{\sqrt{x}} dx = \left| \begin{array}{ll} u' = \frac{1}{\sqrt{x}} & v = \ln x \\ u = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} & v' = \frac{1}{x} \end{array} \right| =$$

$$= \frac{\sqrt{x}}{-\frac{1}{2}} \cdot \ln x - \int \frac{\sqrt{x}}{-\frac{1}{2}} \cdot \frac{1}{x} dx = 2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x} dx = 2\sqrt{x} \ln x - 2 \int x^{-\frac{1}{2}} dx =$$

$$= 2\sqrt{x} \cdot \ln x - 2 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{x} \cdot \ln x - 4\sqrt{x} + C$$

$$14) \int \sin^2 x \, dx = \int \sin x \cdot \sin x \, dx = \left| \begin{array}{ll} u' = \sin x & v = \sin x \\ u = -\cos x & v' = \cos x \end{array} \right| =$$

$$= -\cos x \cdot \sin x + \int \cos x \cdot \cos x \, dx = -\sin x \cdot \cos x + \int (1 - \sin^2 x) \, dx =$$

$$= -\sin x \cdot \cos x + \int dx - \int \sin^2 x \, dx$$

Tento příklad "vede na rovnici"

$$\underbrace{\int \sin^2 x \, dx}_{\text{zadáni}} = -\sin x \cdot \cos x + x - \int \sin^2 x \, dx \quad / + \int \sin^2 x \, dx$$

$$2 \int \sin^2 x \, dx = x - \sin x \cdot \cos x$$

/:2

$$\int \sin^2 x \, dx = \frac{x - \sin x \cdot \cos x}{2} + C$$

Tento příklad lze řešit i substituční metodou (viz soubor Substituční metoda, př. 12)